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蕴涵算子族及其应用

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摘 要 提出了模糊蕴涵算子族的新概念,给出了两族蕴涵算子:$L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$族算子与$L-\lambda-G(\lambda\in[0,1])$族算子.$L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$族算子包括 Lukasiewicz(简称$R_{lu}$)算子与$R_0$算子,$L-\lambda-G(\lambda\in[0,1])$族算子包括$R_{lu}$算子与 Gödel($R_G$)算子.重点讨论了$L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$族算子的伴随算子及其正则性.结果表明,在蕴涵算子族$L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$中,只有$R_{Lu}$算子与$R_0$算子有伴随算子且具有正则性,从而说明这两种算子是较理想的蕴涵算子.最后讨论了其应用,同时提出了命题的置信区间及其可信度的新概念.

关键词 蕴涵算子族;伴随算子;正则性;置信区间;可信度
中图法分类号 TP301

Families of Implication Operators and Their Application

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Abstract In the paper the new concept of family of fuzzy implication operators are introduced, and two new families of fuzzy implication operators are given, which are denoted by $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$ and $L-\lambda-G(\lambda\in[0,1])$. Lukasiewicz operator R_{lu} and operator R_0 are included in $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$, Lukasiewicz operator R_{lu} and operator Gödel (simple denoted R_G) are included in $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$. We mainly discuss regularity of $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$ and the residua of $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$ with its t-norms. The result indicates that only Lukasiewicz operator R_{lu} and operator R_0 in $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$ have residual t-norms and satisfy regularity. Consequently, this two operators are ideal. Finally, the applications of $L-\lambda-R_0\left(\lambda\in\left[\frac{1}{2},1\right]\right)$ in fuzzy reasoning are investigated. In addition, the new concept of believable interval and believable degree of proposition is presented.

Keywords family of implication operators; residual operators; regularity; believable interval; believe degree

收稿日期:2005-04-25;修改稿收到日期:2006-09-21.本课题得到山东省自然科学基金(Y2003A01)资助.**张兴芳**,女,1957 年生,教授,主要研究领域为模糊推理、模糊逻辑、模糊信息处理等.E-mail: zxf@lcu.edu.cn; zhangxingfang2005@126.com.**孟广武**,男,1958 年生,教授,主要研究领域为模糊拓扑、模糊推理、模糊信息处理等.**张安英**,女,1981 年生,硕士研究生,主要研究方向为模糊推理及模糊逻辑.

1 引 言

在模糊逻辑中，主要包括非$\neg$($\neg a=a\to 0$)、上确界$\vee$($a\vee b=\max\{a,b\}$)与蕴涵$\to$三个算子. 人们对于非算子$\neg$及上确界算子$\vee$的定义基本一致，但是，蕴涵算子$\to$的定义却有很多种[1-4]. 至于哪一种最好尚无定论，不过，人们公认便于计算应用又具有相伴随的左连续的三角模且具有正则性[5]的蕴涵算子较好(因为以此建立的逻辑系统具有完备性). 本文提出了蕴涵算子族的新概念，给出了两族蕴涵算子分别称为$L-\lambda-R_0$($\lambda\in\left[\frac{1}{2},1\right]$)族算子与$L-\lambda-G$($\lambda\in[0,1]$)族算子；重点讨论了$L-\lambda-R_0$($\lambda\in\left[\frac{1}{2},1\right]$)族算子，它包括 Lukasiewicz 算子$R_{Lu}$与$R_0$算子. 我们讨论了$L-\lambda-R_0$($\lambda\in\left[\frac{1}{2},1\right]$)族算子的伴随算子及正则性. 结果表明，在蕴涵算子族$L-\lambda-R_0$($\lambda\in\left[\frac{1}{2},1\right]$)中，只有$R_{Lu}$与$R_0$算子具有相伴随的三角模且满足正则性，从而说明这两种算子是较理想的模糊蕴涵算子.

任意给定一个模糊命题(即$F(S)$[4]中的公式)，运用不同的蕴涵算子计算公式的真值一般不相等，甚至误差很大，这在模糊推理的实际应用中，有一定的冒险性，为此本文引入了模糊命题的置信区间及可信度的概念，并通过实例说明了本文给出的两族模糊蕴涵算子和模糊命题的置信区间及可信度概念的重要意义.

限于篇幅，另文再讨论$L-\lambda-G$($\lambda\in[0,1]$)族算子的性质.

2 准 备

定义 1[5]. 设$\otimes:[0,1]^2\to[0,1]$是二元函数，如果当$a,b,c\in[0,1]$时，

(1) $a\otimes b=b\otimes a$；

(2) $(a\otimes b)\otimes c=a\otimes(b\otimes c)$；

(3) $a\otimes 1=a$；

(4) 若$b\le c$，则$a\otimes b\le a\otimes c$.

则称$\otimes$为$[0,1]$上的三角模，简称t-模.

定义 2. 设$\otimes$是$[0,1]$上的t-模，R：$[0,1]^2\to[0,1]$是二元函数. 若$a\otimes b\le c$当且仅当$a\le R(b,c)$，$a,b,c\in[0,1]$，则称R为与$\otimes$相伴随的蕴涵算子，称$(\otimes,R)$为伴随对.

以下$R(a,b)$也常记为$a\to b$.

定义 3[5]. 设$\to$是$[0,1]$上的二元运算，如果$\to$满足下列性质：

(1) $b\to c=1$当且仅当$b\le c$；

(2) $a\le b\to c$当且仅当$b\le a\to c$；

(3) $a\to(b\to c)=b\to(a\to c)$；

(4) $1\to c=c$；

(5) $b\to\bigwedge\limits_{i\in I}c_i=\bigwedge\limits_{i\in I}(b\to c_i)$，$(\bigvee\limits_{i\in I}b_i)\to c=\bigwedge\limits_{i\in I}(b_i\to c)$；

(6) $b\to c$关于c单调递增，关于b单调递减.

则称模糊蕴涵算子$\to$满足正则性.

3 蕴涵算子族

模糊蕴涵算子$R(x,y)=x\to y$是定义在$[0,1]\times[0,1]$上的二元函数，其图像是曲面. 由于模糊逻辑是经典逻辑的推广，自然它应经过 4 个点(0,0,1)，(1,0,0)，(1,1,1)与(0,1,1). 由于经过这 4 个点的曲面无穷多，所以模糊蕴涵算子有很多种. 哪一种模糊蕴涵算子最优还不好定论. 不过，现今人们公认简便易行、满足左连续、正则性且具有伴随的三角模的模糊蕴涵算子较好. 为此下面借助几何图形直观地提出下列两族模糊蕴涵算子.

定义 4. 任意$\lambda\in\left[\frac{1}{2},1\right]$，定义模糊蕴涵算子$\to$如下：

$$x=y\to z=\begin{cases}1, & y\le z\\ 1-y+(2\lambda-1)z, & z+y<1\text{ 且 }y>z,\\ (1-2\lambda)y+z+2\lambda-1, & z+y\ge 1\text{ 且 }y>z\end{cases}$$

$$(y,z)\in[0,1]\times[0,1].$$

特别地，$\lambda=1,\frac{1}{2}$时分别对应R_{Lu}及R_0算子[3]，因此称它为$R_{L-\lambda-R_0}$算子，又称这些算子的全体为模糊蕴涵算子族$L-\lambda-R_0$，$\lambda\in\left[\frac{1}{2},1\right]$.

$x=R(y,z)=y\to z$的图像如图 1 所示.

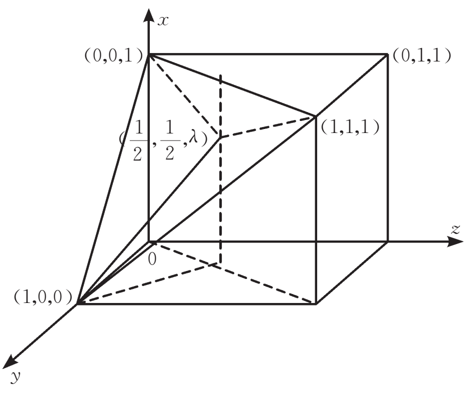

图 1

定义 5. 设→是[0,1]上的蕴涵算子,任意 $\lambda\in[0,1]$,在[0,1]上定义二元函数→如下:

$$x=y\rightarrow z=\begin{cases}1, & y\le z\\ \lambda(1-y)+z, & y>z\end{cases},\ y,z\in[0,1].$$

特别地,$\lambda=1,0$ 时分别为 R_{Lu} 及 R_G 算子,因此称它为 $R_{L-\lambda-G}$ 蕴涵算子. 又称这些算子的全体为蕴涵算子族 $L-\lambda-G$, $\lambda\in[0,1]$.

限于篇幅,下文仅讨论模糊蕴涵算子族 $L-\lambda-R_0$ $\left(\lambda\in\left[\frac{1}{2},1\right]\right)$的性质.

定理 1. 算子⊗:

$$x\otimes y=\begin{cases}0, & x\le 1-y\\ \left.\begin{cases}\dfrac{x+y-1}{2\lambda-1}, & x<1-2(\lambda-1)y\\ y, & 1+2(\lambda-1)y\le x\le 1\end{cases}\right\}, & y<\dfrac{1}{2}\\ \left.\begin{cases}\dfrac{x+y-1}{2\lambda-1}, & x<2\lambda(1-y)\\ (2\lambda-1)(y-1)+x, & 2\lambda(1-y)<x\le 2(1-\lambda)y+2\lambda-1\\ y, & 2(1-\lambda)y+2\lambda-1<x\le 1\end{cases}\right\}, & y\ge\dfrac{1}{2}\end{cases},\ x>1-y$$

满足 $x\otimes y\le z$ 当且仅当 $x\le y\rightarrow z$.

证明. 这里

$$x=R(y,z)=y\rightarrow z$$
$$=\begin{cases}1, & y\le z\\ 1-y+(2\lambda-1)z, & z+y<1 \text{ 且 } y>z\\ (1-2\lambda)y+z+2\lambda-1, & z+y\ge 1 \text{ 且 } y>z\end{cases}.$$

把 y 看作定数,将 $x=y\rightarrow z$ 看作 z 的一元函数如图 2 所示.

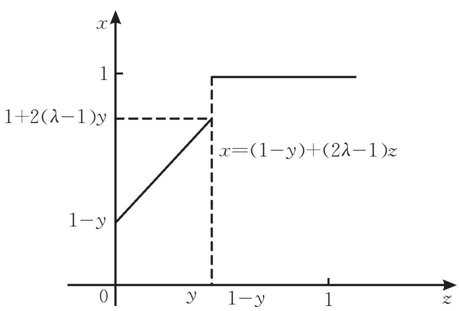

图 2

当 $0\le y<\frac{1}{2}$时,

根据 $x\otimes y\le z$ 当且仅当 $x\le y\rightarrow z$,得到

当 $0\le x\le 1-y$ 时,$\forall z$, $y\rightarrow z\ge 1-y\ge x$,只有 $x\otimes y=0$,才能有 $x\otimes y\le z$, $\forall z$.

当 $1-y\le x\le 2(1-\lambda)y+2\lambda-1$ 时,解 $x\le 1-y+(2\lambda-1)z$ 得到$\frac{x+y-1}{2\lambda-1}\le z$,于是定义 $x\otimes y=\frac{x+y-1}{2\lambda-1}$.

当 $2(1-\lambda)y+2\lambda-1\le x\le 1$ 时,若$x\le y\rightarrow z=1$,则 $y\le z$,所以定义 $x\otimes y=y$.

当 $y\ge\frac{1}{2}$时,$x=y\rightarrow z$,如图 3 所示.

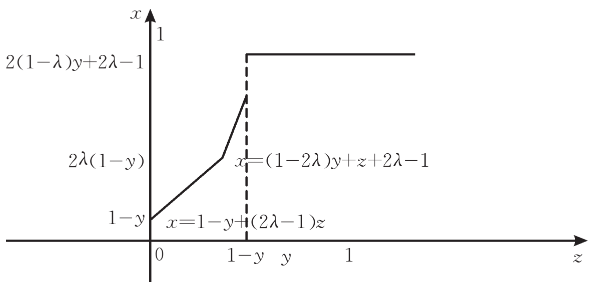

图 3

根据 $x\otimes y\le z$ 当且仅当 $x\le y\rightarrow z$,得到:

当 $0\le x\le 1-y$ 时,与 $y\le\frac{1}{2}$时同理 $x\otimes y=0$;

当 $1-y\le x\le 2\lambda(1-y)$时,与 $y\le\frac{1}{2}$时同理 $x\otimes y=\frac{x+y-1}{2\lambda-1}$;

$2\lambda(1-y)\le x\le 2(1-\lambda)y+2\lambda-1$,解 $x\le(1-2\lambda)y+z+2\lambda-1$ 得到 $x+(1-y)(1-2\lambda)\le z$,于是定义

$$x\otimes y=x+(1-y)(1-2\lambda).$$

当 $2(1-\lambda)y+2\lambda-1<x\le 1$ 时,若 $x\le y\rightarrow z=1$ 则 $y\le z$,所以定义 $x\otimes y=y$.

可以逐条验证以上推理都是可逆的,即如果按定理中的方法求 $x\otimes y$,则 $x\otimes y\le z$ 当且仅当 $x\le y\rightarrow z$ 成立. 证毕.

显然上述定理给出的⊗算子不是三角模. 故蕴涵算子 $R_{L-\lambda-G}$ 当 $\lambda\in\left(\frac{1}{2},1\right)$时没有相伴随的三角模.

当 $\lambda=\frac{1}{2}$,1 时,$R_{L-\lambda-G}$有相伴随的三角模(参见文献[4]).

定理 2. 当 $\lambda\in\left[\frac{1}{2},1\right]$时，蕴涵算子 $R_{L-\lambda-R_0}$ 满足正则性质(i)，(iv)，(v)及(vi)；当 $\lambda=\frac{1}{2}$，1 时蕴涵算子 $R_{L-\lambda-R_0}$ 还满足正则性质(ii)及(iii).

证明. (i)由 $L-\lambda-R_0$，$\lambda\in\left[\frac{1}{2},1\right]$的定义，易见 $b\leq c$ 当且仅当 $b\to c=1$.

(iv)

$$1\to c=\begin{cases}1, & c=1\\ -2\lambda\times1+1+c-1+2\lambda=c, & c<1\end{cases},$$

故 $1\to c=c$.

(v) ① 若 $b\leq\bigwedge_{i\in I}c_i$，则对任意的 $i\in I$ 都有 $b\leq c_i$，从而

$$b\to c_i=1,\text{则}\bigwedge_{i\in I}(b\to c_i)=1.$$

已知 $b\to\bigwedge_{i\in I}c_i=1$，所以 $b\to\bigwedge_{i\in I}c_i=\bigwedge_{i\in I}(b\to c_i)$.

② 若 $b>\bigwedge_{i\in I}c_i$ 且 $b+\bigwedge_{i\in I}c_i\leq1$，则存在 $I_0\subset I$，使得对任意 $i_0\in I_0$ 有 $b>c_{i_0}$ 且 $b+c_{i_0}\leq1$. 从而

$$\bigwedge_{i\in I}(b\to c_i)=\bigwedge_{i\in I_0}(b\to c_{i_0})=\bigwedge_{i\in I_0}[(2\lambda-1)c_{i_0}-b+1]$$
$$=(2\lambda-1)\bigwedge_{i\in I_0}c_{i_0}-b+1=b\to\bigwedge_{i\in I}c_i.$$

③ 若 $b>\bigwedge_{i\in I}c_i$ 且 $b+\bigwedge_{i\in I}c_i>1$ 则对任意的 $i\in I$，有$b+c_i>0$. 从而

$$\bigwedge_{i\in I}(b\to c_i)=\bigwedge_{i\in I}[(1-2\lambda)b+c_i+2\lambda-1]$$
$$=(1-2\lambda)b+c_i+(2\lambda-1)=b\to\bigwedge_{i\in I}c_i\ .$$

(vi) 固定 $b_0\in[0,1]$，任取 $c_1,c_2\in[0,1]$，且$c_1<c_2$，则

$$b_0\to c_1=\begin{cases}1, & b_0\leq c_1\\ \begin{cases}2\lambda c_1-(b_0+c_1)+1, & b_0+c_1\leq1\\ -2\lambda b_0+b_0+c_1-1+2\lambda, & b_0+c_1>1\end{cases}, & b_0>c_1;\end{cases}$$

$$b_0\to c_2=\begin{cases}1, & b_0\leq c_2\\ \begin{cases}2\lambda c_2-(b_0+c_2)+1, & b_0+c_2\leq1\\ -2\lambda b_0+b_0+c_2-1+2\lambda, & b_0+c_2>1\end{cases}, & b_0>c_2.\end{cases}$$

① 当 $b_0\leq c_2$时，显然有 $b_0\to c_1\leq b_0\to c_2$；

② 当 $b_0>c_2$时

a) 若 $b_0+c_2\leq1$，$b_0+c_1<1$，

$$b_0\to c_2-b_0\to c_1=[2\lambda c_2-(b_0+c_2)+1]-[2\lambda c_1-(b_0+c_1)+1]=(2\lambda-1)(c_2-c_1)\geq0$$

b) 若 $b_0+c_1>1$，$b_0+c_2>1$，

$$b_0\to c_2-b_0\to c_1=(-2\lambda b_0+b_0+c_2-1+2\lambda)-(-2\lambda b_0+b_0+c_1-1+2\lambda)=c_2-c_1>0$$

c) 若 $b_0+c_1\leq1$ 但 $b_0+c_2>1$，

$$b_0\to c_2-b_0\to c_1=(-2\lambda b_0+b_0+c_2-1+2\lambda)-[2\lambda c_1-(b_0+c_1)+1]=2(1-\lambda)(1-b_0)+c_2-(2\lambda-1)c_1\geq2(1-\lambda)c_1+c_2-(2\lambda-1)c_1=(3-4\lambda)c_1+c_2.$$

因为$\frac{1}{2}\leq\lambda\leq1$，所以$-1\leq-4\lambda\leq-2$，$-1\leq3-4\lambda\leq1$，$0<c_2-c_1\leq(3-4\lambda)c_1+c_2\leq c_1+c_2$.

综上所述，对任意一种情况，只要 $c_1<c_2$，都有 $b_0\to c_2\geq b_0\to c_1$，从而 $b\to c$ 关于 c 单调递增.

用类似的方法可证明 $b\to c$ 关于 b 单调递减.

总之，当 $\lambda\in\left[\frac{1}{2},1\right]$时，蕴涵算子 $R_{l-\lambda-R_0}$ 满足正则性质(i)，(iv)(v)及(vi).

当 $\lambda=\frac{1}{2}$，1 时满足正则性质(ii)及(iii)的证明参见文献[4].

注. $L-\lambda-R_0$ 族蕴涵算子当 $\lambda\in\left(\frac{1}{2},1\right)$时不满足正则性质(ii)及(iii).

例如，$\forall\lambda\in\left(\frac{1}{2},1\right)$，取 $c=0.1$，$b=0.91$，$a=0.1+0.09(2\lambda-1)$. 由于 $a>c$ 且 $a+c<1$，$b>c$ 且 $b+c>1$，则

$$\begin{aligned}b\to c&=(1-2\lambda)b+c+2\lambda-1\\&=(2\lambda-1)(1-0.91)+0.1\\&=0.09(2\lambda-1)+0.1=a,\\a\to c&=1-a+(2\lambda-1)c\\&=0.9-0.09(2\lambda-1)+0.1(2\lambda-1)\\&=0.9+0.01(2\lambda-1)<0.91=b.\end{aligned}$$

$a\to(b\to c)=1$，但 $b\to(a\to c)<1$.

由此说明 $L-\lambda-R_0$ 族蕴涵算子当 $\lambda\in\left(\frac{1}{2},1\right)$时，不满足正则性质(ii)和(iii).

4 应用、置信区间与可信度

$L-\lambda-R_0$族的任一算子都是线性算子，计算简单，便于应用. 一方面它给实际应用提供了一些选择的模型；另一方面，选取带参数的算子族会使结论更加明朗化，决策尽量减少盲动性，看下面的例子.

例 1. 公安人员审查一件盗窃案，已掌握的事实有：

(1) 八成是 A 或 B 作的案；

(2) 若是 A 或 B 作的案，则作案的时间根本不

可能发生在午夜之前；

(3) 若 B 证词仅八成属实，午夜时屋内灯光一成未灭；

(4) 若 B 证词不属实，则案件发生在午夜之前的可能性极大；

(5) 据多方调查，午夜时屋内灯光确实灭了；

问：谁最有可能是罪犯？

为了便于进行模糊推理演算，我们先将已知的事实进行量化和符号化. 设

p：是 A 作的案；

q：是 B 作的案；

r：作案时间发生在午夜之前；

s：B 证词属实；

t：午夜时灯光未灭.

根据前提条件，选取 $L-\lambda-R_0$ 蕴涵算子族，依据模糊推理规则，推理过程可进行如下：

$$T(s)=0.8, T(t)=1-T(\neg t)=0.1.$$

(1) $s\to t$

$$\begin{aligned}T(s\to t)&=1-T(s)+(2\lambda-1)T(t)\\&=1-0.8+0.1(2\lambda-1)=0.2\lambda+0.1.\end{aligned}$$

(2) $\neg t$

$T(\neg t)=1-T(t)=1-0.1=0.9$(前提引入).

(3) $\neg s$((1),(2)拒取式)

$$\begin{aligned}T(\neg t\to\neg s)&=T(s\to t)=0.2\lambda+0.1,\\T(\neg s)&=T(\neg t\to\neg s)\wedge T(\neg t)\\&=(0.2\lambda+0.1)\wedge 0.9\\&=0.2\lambda+0.1.\end{aligned}$$

(4) $\neg s\to r$

$$T(\neg s\to r)=0.9.$$

(5) r((3),(4)假言推理)

$$\begin{aligned}T(r)&=T(\neg s\to r)\wedge T(\neg s)\\&=0.9\wedge(0.2\lambda+0.1)=0.2\lambda+0.1.\end{aligned}$$

(6) $p\to\neg r$

$T(p\to\neg r)=1.0$(前提引入).

(7) $\neg p$((5),(6))

$$\begin{aligned}T(\neg p)&=T(p\to\neg r)\wedge T(r)\\&=(0.1+0.2\lambda)\wedge 1\\&=0.2\lambda+0.1,\\T(P)&=1-T(\neg p)=1-(0.2\lambda+0.1)\\&=0.9-0.2\lambda.\end{aligned}$$

根据推理，A 作案的可能性是 $0.9-0.2\lambda$.

(8) $p\vee q$ ：$T(p\vee q)=0.8$(前提引入).

(9) q((7),(8)析取三段论)

$T(q)=T(p\vee q)\wedge T(\neg p)=0.1+0.2\lambda.$

根据推理，B 作案的可能性是 $0.1+0.2\lambda$.

下面我们通过图 4 分析对比 A 与 B 作案的可能性.

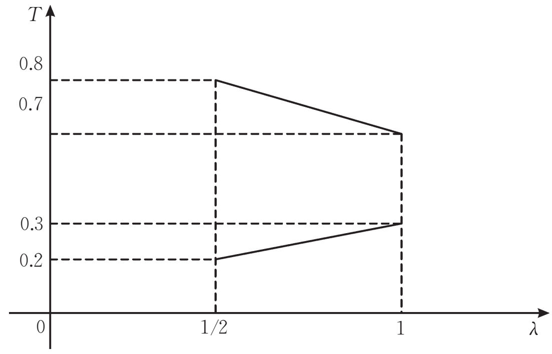

图 4

从图 4 可以看出，A 作案的可能性大.

上例说明，选取不同的模糊蕴涵算子进行推理时得到的结论会有很大的差别.

从上例我们看到，选取蕴涵算子族进行推理，往往结论是一个定义在[0,1]上的 λ 的连续函数，为此我们引入下列概念.

定义 6. 设运用蕴涵算子族进行推理的结论 P(即命题)是一个定义在[0,1]上的关于 λ 的连续函数 $f(\lambda)$. 我们称 $f(\lambda)$ 的值域为结论 P 的置信区间. 设 P 的置信区间的长度为 l，则称 $k=1-l$ 为结论 P 的可信度.

根据上述定义，A 作案的置信区间是[0.7,0.8]，可信度是 0.9；

B 作案的置信区间是[0.2,0.3]，可信度是 0.9.

若选取蕴涵算子族进行模糊推理，会使得到的信息更加明朗化，这样可使决策减少盲动性.

5 结 论

本文给出了模糊蕴涵算子族

$$x=y\to z=\begin{cases}1, & y\le z\\1-y+(2\lambda-1)z, & z+y<1 \text{ 且 } y>z\\(1-2\lambda)y+z+2\lambda-1, & z+y\ge 1 \text{ 且 } y>z\end{cases},$$

$$(y,z)\in[0,1]\times[0,1], \lambda\in\left[\frac{1}{2},1\right]$$

(记为 $L-\lambda-R_0$)

和

$x=y\to z=\begin{cases}1, & y\le z\\\lambda(1-y)+z, & y>z\end{cases}$，$y,z\in[0,1]$，$\lambda\in[0,1]$(记为 $L-\lambda-G$). 通过两个定理得到模糊蕴涵算子族 $L-\lambda-R_0$ 的性质：当 $\lambda\in\left(\frac{1}{2},1\right)$时，蕴涵算

子 $R_{L-\lambda-G}$没有相伴随的三角模，满足正则性质(i)，(iv)，(v)和(vi)，但不满足正则性质(ii)及(iii)(见上面). 当 $\lambda=\frac{1}{2}$及 1 时，$R_{L-\lambda-G}$分别为 R_0及 R_{lu}算子，它们都有相伴随的三角模，且满足正则性，因此是较理想的算子.

本文还通过一个公安人员审查一件盗窃案的实例，说明了模糊蕴涵算子族 $L-\lambda-R_0$在近似推理中的具体应用，同时引入了推理结果的置信区间及可信度的概念.

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Background

Negative operator $\neg$, disjunction operator $\vee$ and implication operator $\rightarrow$ are three main operators in fuzzy logic, which are unary, binary and binary operators, respectively. The definitions of $\neg$ and $\vee$ are basic consistent, that is, for all $a\in[0,1]$, $\neg a=1-a$, $a\vee b=\max\{a,b\}$).

However, the definition of operator $\rightarrow$ is different in different logic systems. There are many definition for $\rightarrow$, But it is no last word that which definition is best. It is thought that implication operators with left-continuous residual t-norm and regularity are suitable to establish complete fuzzy logical systems. It is well known that Compositional Rule of Inference (Briefly, CRI) introduced by Zadeh plays an important role in fuzzy control. The result of fuzzy reasoning is connected with the selection of fuzzy implication operators. Selection of the implication operators should consider not only actual problems but also simplicity of calculations and applications. It is clear that different selection will retain a different result in fuzzy reasoning, even the difference is very large. Based on this reason, the conception of the family of fuzzy implication operators (or fuzzy implication operators with parameters) was introduced in this paper, we propose two new families of fuzzy implication operators, which are denoted by $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$, and $L-\lambda-G,\lambda\in[0,1]$ respectively. Since these operators are linear, their calculation steps are simpler. We observe that $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$ includes Lukasiewicz operator R_{lu} and R_0, $L-\lambda-G,\lambda\in[0,1]$ includes R_{lu} and R_G. We mainly discuss regularity of $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$ and the residua of $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$ with its t-norms. The result indicates that only Lukasiewicz operator R_{lu} and operator R_0 in $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$ have residual t-norms and satisfy regularity. Consequently, this two operators are ideal. Finally, the applications of $L-\lambda-R_0,\lambda\in\left[\frac{1}{2},1\right]$ in fuzzy reasoning are investigated. In addition, the new concept of believable interval and believable degree of proposition is presented.

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